An Electronic Mach-Zehnder Quantum Eraser

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We propose an electronic quantum eraser in which electrons are injected into a mesoscopic conductor in the quantum Hall regime. The conductor is composed of a two-path interferometer, an electronic analog of the optical Mach-Zehnder interferometer, and a quantum point contact detector capacitively coupled to the interferometer. While the interference of the output current at the interferometer is suppressed by the *which-path* information, we show that the which-path information is erased and the interference reappears in the cross correlation measurement between the interferometer and the detector output leads. We also investigate a modified setup in which the detector is replaced by a two-path interferometer. We show that the distinguishability of the path and the visibility of joint detection can be controlled in a continuous manner and satisfy a complementarity relation for the entangled electrons.

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Complementarity in quantum theory is well described in a double-slit interferometer. In a two-path interferometer with a *which-path* (WP) detector, the observation of interference and acquisition of WP information are mutually exclusive [1, 2, 3, 4]. Feynman argued that any attempt to extract WP information would disturb the motion of the injected particle and wash out the interference pattern [1]. That is, if one can fully determine the path, then there is no interference at all that results from the uncertainty of the phase of the incident particle [3]. The interference is lost even if one does not actually readout the WP detector, as far as there is a mere possibility of carrying out the measurement. However, the loss of interference need not be irreversible if the WP detector itself is a quantum system. In particular, it has been proposed [4, 5] that in some cases the loss of interference may be simply because of the quantum correlation of the interferometer with the WP detector. In this case the WP information can be *erased* by a suitable measurement on the detector. This 'quantum eraser' has been realized in various setups by using entangled photons [6].

On the other hand, mesoscopic physics is evolving into a stage where understanding the measurement process is becoming important. Indeed, WP detection in quantum interferometers has been achieved by using mesoscopic conductors [7, 8, 9]. In these experiments, a quantum point contact (QPC) was used as a WP detector by probing the charge of a single electron at a nearby quantum dot (QD). There have been numerous theoretical studies on how detection of a charge suppresses the interference [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. In many theoretical works on mesoscopic WP detection, suppression of interference due to quantum correlation is identified with the back-action dephasing caused by the disturbance of the electronic motion by the detector [3]. However, in principle, a quantum erasure is possible also for mesoscopic systems unless the suppression of the interference is not accompanied by irreversible phase randomization. There was a theoretical proposal for a mesoscopic quantum eraser by exploiting the internal degree of freedom for the QD state [22]

In this Letter, we propose a new idea for a quantum eraser composed of an electronic Mach-Zehnder interferometer (MZI) [23, 24] and a QPC detector. Two different types of quantum erasers are considered. In the first setup, an electronic beam splitter plays the role of the WP detector (Fig. 1). The MZI and the beam splitter are capacitively coupled in a way that the WP information is stored in the scattering phase because of Coulomb interaction. An electronic analog of the beam splitter can be made by using a QPC at high magnetic fields [25]. An electronic MZI has recently been realized by using quantum Hall edge states [24, 26, 27]. Therefore it is possible to achieve the arrangements of Figs. (1,2). Without the detector, the output currents at lead α and at β will show interference as a function of the phase (φ) enclosed by the loop of the MZI. This phase can be controlled either by an external magnetic field or by an electrical gate [24, 26, 27]. Coulomb interaction between the electrons of the interferometer and the detector leads to a change in the electronic trajectory (denoted by the dashed lines), when both electrons are transmitted. This modification of the trajectory gives partial or full WP information for the electron in the MZI and suppresses the interference. Interestingly, the WP information can be erased by measuring the joint-detection probability of two electrons at lead A ($\in \alpha,\beta$) and lead B ($\in \gamma,\delta$). Furthermore, in the second setup where the detector is also made of a MZI (Fig. 2), the path distinguishability and the visibility of the joint-detection probability can be controlled by modulating the phase in the detector. In other words, one can choose the 'particle-like' or the 'wave-like' behavior by an appropriate manipulation of the detector. It is important to note that our quantum eraser exploits only the charge degree of freedom which is relatively easy to control in mesoscopic devices.

Let us consider a two-electron injection process (one from the MZI and the other from the detector) in the first setup (Fig. 1). For convenience, two types of electron creation operators are defined, namely $c_x^\dagger$ and $b_x^\dagger$. The operators $c_x^\dagger$ and $b_x^\dagger$ create an electron at lead x and at the intermediate regions, respectively. The beam splitter, BS-i, is characterized by the scattering matrix S_i ($i = 1,2,3$)

$$S_i = \begin{pmatrix} r_i & t_i' \\ t_i & r_i' \end{pmatrix} , \tag{1a}$$

which transforms the electron operators as

$$(c_{\bar\alpha}^\dagger \;\; c_{\bar\beta}^\dagger) = (b_\alpha^\dagger \;\; b_\beta^\dagger) S_1 , \tag{1b}$$
$$(b_{\bar\alpha}^\dagger \;\; b_{\bar\beta}^\dagger) = (c_\alpha^\dagger \;\; c_\beta^\dagger) S_2 , \tag{1c}$$
$$(c_{\bar\gamma}^\dagger \;\; c_{\bar\delta}^\dagger) = (c_\gamma^\dagger \;\; c_\delta^\dagger) S_3 . \tag{1d}$$

Before interaction at the intermediate region, the two-electron state can be written as

$$|\Psi_0\rangle = c_{\bar\alpha}^\dagger c_{\bar\gamma}^\dagger |0\rangle = (r_1 b_\alpha^\dagger + t_1 b_\beta^\dagger) \otimes (r_3 c_\gamma^\dagger + t_3 c_\delta^\dagger)|0\rangle \tag{2}$$

where $|0\rangle$ is the ground state without an electron injection into the conductor. Coulomb repulsion modifies the trajectory of the two electrons when both electrons are transmitted (dashed lines). Here it is assumed that the Coulomb interaction affects only the trajectory of the two electrons. The inelastic scattering is neglected. This modification of the trajectory gives an additional phase shift $\Delta\phi$ given as

$$\Delta\phi = 2\pi H \Delta A / \Phi_0 , \tag{3}$$

where H and ΔA stand for the external magnetic field and the area enclosed by the change of the trajectory resulting from the interaction (denoted by shaded regions of Figs.(1,2)). Φ_0 is the flux quantum of an electron, $\Phi_0 = hc/e$. As a result, the two-electron state upon a scattering can be written as

$$|\Psi\rangle = (r_1 r_3 b_\alpha^\dagger c_\gamma^\dagger + r_1 t_3 b_\alpha^\dagger c_\delta^\dagger + t_1 r_3 b_\beta^\dagger c_\gamma^\dagger + t_1 t_3 e^{i\Delta\phi} b_\beta^\dagger c_\delta^\dagger)|0\rangle , \tag{4}$$

or can be simplified as

$$|\Psi\rangle = (r_1 b_\alpha^\dagger \chi_r^\dagger + t_1 b_\beta^\dagger \chi_t^\dagger)|0\rangle . \tag{5a}$$

The operators $\chi_r^\dagger$ and $\chi_t^\dagger$ create the detector states depending on whether the electron in the MZI is reflected or transmitted, respectively. These operators are written as

$$\chi_r^\dagger = r_3 c_\gamma^\dagger + t_3 c_\delta^\dagger , \tag{5b}$$
$$\chi_t^\dagger = r_3 c_\gamma^\dagger + t_3 e^{i\Delta\phi} c_\delta^\dagger . \tag{5c}$$

Eq. (5) describes an entanglement between the MZI and the detector. One can see that $\chi_r^\dagger \neq \chi_t^\dagger$ because of the phase factor $e^{i\Delta\phi}$, and the extent of the entanglement can be controlled by changing H or ΔA. In this way, the WP information in the MZI is stored in the detector state.

The interference of single electrons in the MZI will be reflected in the probability of finding an electron at lead A ($\in \alpha, \beta$),

$$P_A = \langle \Psi | c_A^\dagger c_A | \Psi \rangle . \tag{6}$$

The evaluation can be done with the help of Eqs. (1,5). The evaluation gives

$$\begin{aligned} P_\alpha &= 1 - P_\beta \\ &= R_1 R_2 + T_1 T_2 + 2|\nu| \sqrt{R_1 T_1 R_2 T_2} \cos(\varphi - \phi_\nu) , \end{aligned} \tag{7}$$

where $T_i = |t_i|^2$ and $R_i = |r_i|^2$ correspond to the transmission and the reflection probability, respectively, for beam splitter BS-i. The overlap of the detector states $\nu \equiv \langle 0 | \chi_t \chi_r^\dagger | 0 \rangle$ is a quantitative measure of the WP information and $\phi_\nu \equiv \arg \nu$. A small overlap indicates nearly orthogonal detector states which distinguishes the path of an electron in the MZI. The phase φ enclosed by the loop of the MZI is given as $\varphi = \arg(t_1) + \arg(t_2') - \arg(r_1) - \arg(r_2)$.

Eq. (7) shows the relation between the interference fringe and the WP information stored in the detector. If the two detector states $\chi_r^\dagger|0\rangle$ and $\chi_t^\dagger|0\rangle$ are orthogonal (that is $\nu = 0$), then the electron in the MZI acquires the complete WP information and the interference disappears. Complete WP information can be obtained for a symmetric BS-3 ($|r_3| = |t_3| = 1/\sqrt{2}$) with $\Delta\phi = \pi$.

Acquisition of the WP information in our setup results from the quantum correlation between the two subsystems. Since the detector itself is a two-state quantum system, the WP information can be erased by a suitable measurement on the detector. Indeed, a joint detection of two electrons (one from a lead of the MZI and the other from a lead of the detector) renders a measurement of $\Delta\phi$ impossible and therefore erases the WP information. For simplicity, our discussion is limited to the simple case in which the two beam splitters of the MZI are symmetric ($R_i = T_i = 1/2$ for $i = 1, 2$). The joint-detection probability P_{AB} denotes the probability of finding an electron at lead A ($\in \alpha, \beta$) and the other electron at lead B ($\in \gamma, \delta$) simultaneously, defined as

$$P_{AB} = \langle \Psi | c_A^\dagger c_A c_B^\dagger c_B | \Psi \rangle . \tag{8}$$

For the state $|\Psi\rangle$ defined in Eq. (5), we find

$$P_{\alpha\gamma} = R_3 [1 + \cos\varphi]/2 , \tag{9a}$$
$$P_{\alpha\delta} = T_3 [1 + \cos(\varphi + \Delta\phi)]/2 , \tag{9b}$$
$$P_{\beta\gamma} = R_3 [1 - \cos\varphi]/2 , \tag{9c}$$
$$P_{\beta\delta} = T_3 [1 - \cos(\varphi + \Delta\phi)]/2 . \tag{9d}$$

This result shows that *the visibility of P_{AB} is not affected by the parameter ν*. That is, the WP information encoded

in the phase shift $\Delta\phi$ is deleted by the joint-detection measurement and the hidden coherence reappears.

Let us now consider how this effect can be verified experimentally for a mesoscopic conductor. At a voltage bias eV for the input leads $\bar{\alpha}$ and $\bar{\gamma}$, we can write the 'entangled' many-body transport state as

$$|\bar{\Psi}\rangle = \prod_{0<E<eV} \left[r_1 b_\alpha^\dagger(E)\chi_r^\dagger(E) + t_1 b_\beta^\dagger(E)\chi_t^\dagger(E) \right] |\bar{0}\rangle \ , \tag{10}$$

where $|\bar{0}\rangle$ stands for the ground state, a filled Fermi sea in all leads at energies $E < 0$. The crucial assumption made here is that the injected electrons from the two sources interact with each other and are transmitted as entangled pairs as illustrated in Fig. 1.

For the state $|\bar{\Psi}\rangle$, the output current at lead A (I_A) is proportional to the probability of finding an electron at this lead, $I_A = (e^2/h)P_A V$, at zero temperature. The zero-frequency cross correlation, S_{AB}, of the current fluctuations, ΔI_A and ΔI_B, is defined as [25]

$$S_{AB} = \int dt \, \langle \bar{\Psi} | \Delta I_A(t) \Delta I_B(0) + \Delta I_B(0) \Delta I_A(t) | \bar{\Psi} \rangle \ , \tag{11}$$

where $\Delta I_A = I_A - \langle I_A \rangle$ and $\Delta I_B = I_B - \langle I_B \rangle$. For $A \in \alpha, \beta$ and $B \in \gamma, \delta$, this cross correlator provides information about the two-particle interactions between the MZI and the detector. After some algebra, one can find the following useful relation:

$$S_{AB} = \frac{2e^2}{h} eV (P_{AB} - P_A P_B) \ . \tag{12}$$

Therefore the values of the single-particle (P_A) and the joint-detection (P_{AB}) probabilities can be obtained by measuring the current and the zero-frequency cross correlation.

For the first setup shown in Fig. 1, the WP information is contained in the detector by the entanglement of the electronic trajectories and is erased by measuring the coincidence count. Next, we consider a modified setup where another beam splitter (BS-4) is inserted in the detector (see Fig. 2) so that the detector itself is also a MZI. In the following, the upper MZI is labeled as "MZI-s" and the lower MZI as "MZI-d". In this geometry, the WP information may be erased, or marked, or partially erased by appropriate control of the detector. The control of the WP information is possible in a continuous manner as a function of the phase (φ_d) enclosed by the loop of the MZI-d.

In addition to the scattering matrices defined in Eq. (1), we need another scattering matrix S_4 to describe the BS-4 introduced in the same as that expressed in Eq. (1). The probability of finding a single electron at lead A, P_A ($A \in \alpha, \beta$), is the same as that of Eq. (6). This is because the quantity ν is not modified by the unitary scattering process at the BS-4. On the other hand, the joint-detection probability is given, for instance, as

$$P_{\alpha\gamma} = |r_1 r_2 u_\gamma + t_1 t_2' v_\gamma|^2 \ , \tag{13}$$

where the coefficient $u_\gamma \equiv r_3 r_4 + t_3 t_4'$ ($v_\gamma \equiv r_3 r_4 + t_3 t_4' e^{i\Delta\phi}$) represents the amplitude of finding an electron at lead γ under the condition that the electron in the MZI-s passes through the upper (lower) path. These coefficients can be controlled through the phase φ_d enclosed by the loop of the detector given as $\varphi_d = \phi_{t_3} + \phi_{t_4'} - \phi_{r_3} - \phi_{r_4}$. For example, $|u_\gamma|$ and $|v_\gamma|$ are given as $|u_\gamma| = |1 + e^{i\varphi_d}|/2$ and $|v_\gamma| = |1 - e^{i\varphi_d}|$, for the symmetric beam splitters in MZI-d ($R_3 = T_3 = 1/2, R_4 = T_4 = 1/2$). The other joint-detection probabilities can be evaluated in a similar way. The two amplitudes, $r_1 r_2 u_\gamma$ and $t_1 t_2' v_\gamma$, (schematically drawn in Fig. 2(b) and (c), respectively) are indistinguishable and lead to the interference fringe for $P_{\alpha\gamma}$ with its visibility given by

$$\mathcal{V} = \frac{2\sqrt{R_1 R_2 T_1 T_2} |u_\gamma| |v_\gamma|}{R_1 R_2 |u_\gamma|^2 + T_1 T_2 |v_\gamma|^2} \ . \tag{14}$$

One can find that $0 \leq \mathcal{V} \leq 1$. This result shows that the visibility of the joint-detection probability can be controlled through the phase φ_d. In other words, the WP information in the MZI-s can be marked ($\mathcal{V} = 0$), erased ($\mathcal{V} = 1$), or partially erased ($0 < \mathcal{V} < 1$), by its entangled twin in the MZI-d.

The observations of the interference pattern and the acquisition of the path information are mutually exclusive. In the following, we quantitatively express this interferometric duality. Visibility, $\mathcal{V}$, of Eq. (14) is a measure of the interference fringe. On the other hand, the amount of the path information can be expressed by the *distinguishability* of the two paths (Fig. 2(b,c)) defined as

$$\mathcal{D} \equiv \frac{\left| R_1 R_2 |u_\gamma|^2 - T_1 T_2 |v_\gamma|^2 \right|}{R_1 R_2 |u_\gamma|^2 + T_1 T_2 |v_\gamma|^2} \ . \tag{15}$$

This number, which is in the range of $0 \leq \mathcal{D} \leq 1$, is a quantitative measure of the knowledge of the paths [28, 29]. Then, one finds the duality relation

$$\mathcal{D}^2 + \mathcal{V}^2 = 1 \ . \tag{16}$$

The duality relation of Eq. (16) can be tested experimentally. The visibility is obtained from the interference pattern of $P_{\alpha\gamma}$ (Eq. (13)). The distinguishability is available in an independent measurement of $|u_\gamma|^2$ and $|v_\gamma|^2$. The quantities $|u_\gamma|^2$ and $|v_\gamma|^2$ are proportional to the output current at lead γ in which the electron at the BS-1 is fully reflected ($T_1 = 0$) or fully transmitted ($T_1 = 1$), respectively. This independent measurement on $\mathcal{V}$ and $\mathcal{D}$ is expected to reveal the duality relation.

The duality relation of Eq. (16) is valid under the assumption that the two injected electrons are described by an ideal pure state. In general, the two-electron

state may be affected by the environment. In this case the duality relation is modified as an inequality such as $\mathcal{D}^2 + \mathcal{V}^2 \leq 1$.

Our discussion on the quantum erasure is based on the interferometer-detector entanglement of the electron trajectories. This kind of entanglement has no optical analogue since the entanglement considered here is formed by the Coulomb interactions that affect the trajectories of the electrons. Further, it is also possible to study the novel two-particle correlation, such as the Bell's inequality test [30], using the entanglement proposed here.

Finally, we add some remarks concerning the experimental feasibility of our proposal, especially about the phase shift $\Delta\phi$ of Eq. (3) induced by the Coulomb interaction. In order to carry out the experiment, it is essential to get $\Delta\phi \sim \pi$. However, it is questionable if two spatially separated ballistic edge channels suffer enough Coulomb interactions leading to $\Delta\phi \sim \pi$. A possible solution to this question is to use two quantum Hall edge states at a filling factor of 2. That is, with appropriate control of the two edge states, one channel is used as an interferometer and the other as a detector. Indeed, it has been shown that the Coulomb interaction between the two edge channels at a filling factor of 2 is strong enough to introduce a phase shift greater than π [27]. An alternative to get a strong Coulomb-interaction-induced phase shift would be to insert a quantum dot in the interferometer as it has been conducted in the earlier WP interferometer [7].

In conclusion, we have proposed a possible realization of the electronic quantum eraser by using a two-path interferometer and a quantum point contact detector in the quantum Hall regime. The Coulomb interaction between the interferometer and the detector induces a phase shift that enables the which-path detection. We have shown that the which-path information can be erased by a joint-detection measurement and then the hidden coherence reappears. Furthermore, it is possible to choose the 'particle-like' or the 'wave-like' behavior by appropriate control of the detector in a modified setup.

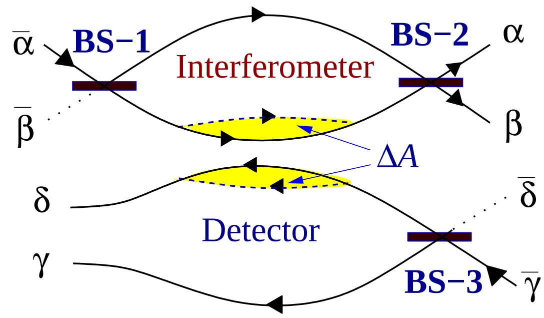

FIG. 1: A schematic diagram of the first setup for an electronic Mach-Zehnder quantum eraser consisting of a Mach-Zehnder interferometer and a QPC detector. The Coulomb interaction between the two electrons modifies their trajectories, giving rise to a change of the areas, ΔA. The interference of the output currents at leads α, β fully or partially vanishes by the which-path information encoded in the phase shift $\Delta\phi$ of Eq. (3). The interference reappears by a joint-detection count (zero-frequency cross correlation) between a lead of the interferometer (α, β) and a lead in the detector (γ, δ).

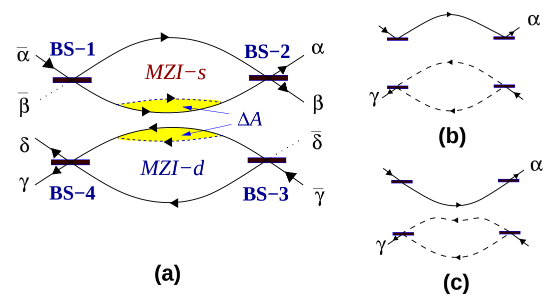

FIG. 2: (a) A schematic diagram of the 2st setup for an electronic Mach-Zehnder quantum eraser consisting of two coupled Mach-Zehnder interferometers with one being used as an interferometer (MZI-s) and the other as a detector (MZI-d). In this geometry, the path information of the MZI-s may be erased or restored by appropriate choices of the phase φ_d in the detector. (b,c) Two indistinguishable processes for joint-detection at leads α and γ.

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